



يمكن استخدام الكتاب ودفتر الملاحظات ولا يسمح باستخدام الهواتف الذكية

الامتحان النهائي لمادة كهرومغناطيسية I

الزمن ساعتان ونصف

أجب عن جميع الأسئلة التالية:

السؤال الأول: (الدرجة 50%)

1- أوجد الشحنة الكهربائية الناتجة من المجال E :

$$E = a_r 12r + a_\theta 5r \sin \theta + a_\phi 10r^2$$

2- أوجد التيار الكهربائي الناتج من الفيض المغناطيسي B والمار في سلك مساحة مقطعه A :

$$B = a_\rho 5\rho z \cos \varphi + a_\phi 10\rho z \sin \varphi$$

3- أوجد زوايا السقوط للمجال E عندما تكون $r = 1$ و $\theta = 20^\circ$ ، ثم أوجد متجه الوحدة.

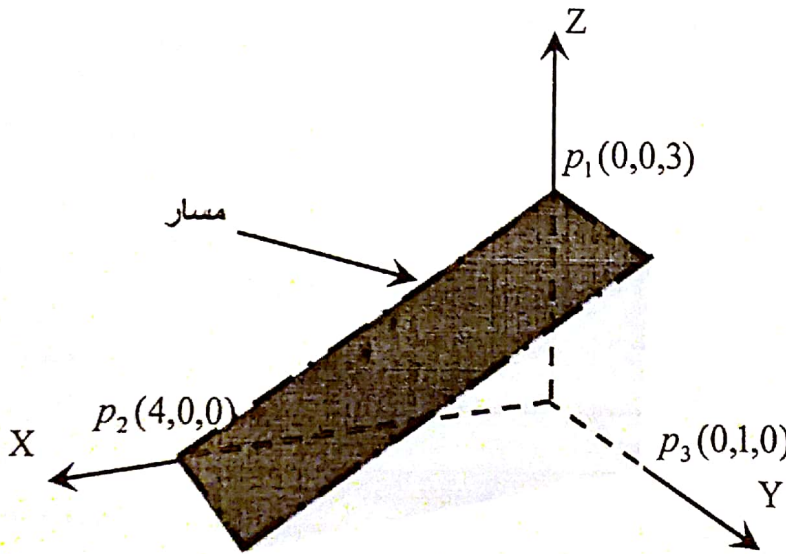
السؤال الثاني: (الدرجة 50%)

1- إذا كان التغير في المجال الكهربائي ∇E يعطى بالعلاقة $\nabla E = 10x a_x + 6a_y$ ، أوجد دالة المجال E .

2- إذا كان المجال الكهربائي يعطى بالعلاقة: $H = Kya_y$ ، أثبت أن نظرية التدرج (Divergence Theorem) صحيحة باستخدام الشكل 1.

3- استخدم المسار الموضح في الشكل 1 لإثبات نظرية ستوكس (Theorem of Stokes).

بالتوفيق والنجاح



الشكل 1

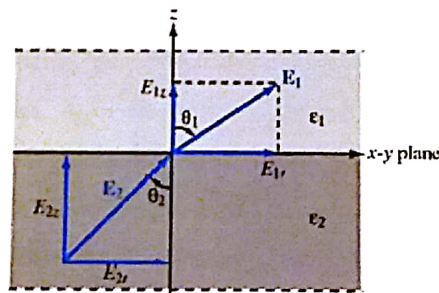
Q1:

- I. If $z = 3 - j5$, Find the value of $\ln(z)$. (5 points)
- II. Express the following complex number in rectangular form $\sqrt{3} e^{j^{3\pi/4}}$ (10 points)

Q2: Given vectors $A = a_x + 2a_y - 3a_z$ and $B = 2a_x - 4a_y$.

- I. Magnitude of A, and the unit vector of A. (5 points)
- II. θ_{AB} , (5 points)
- III. $(A \times a_y) \cdot a_z$ (5 points)

Q3: Find E_1 in the Fig below if $E_2 = 2a_x - 3a_y + 3a_z$ V/m if $\epsilon_1 = 2\epsilon_0$, $\epsilon_2 = 8\epsilon_0$, and the boundary is charge free. (15 points)



Q4: Suppose a scalar function in the cylindrical domain is given as :

$$p = \rho^2 \sin \varphi + 2\rho z$$

- i. Find the Gradient of P, Show your work, is the Gradient a scalar or a vector? (7 points)
- ii. Now find the curl of the above gradient, what do you observe? (7 points)

$$\text{curl } \mathbf{F} = \frac{\mathbf{a}_1}{h_2 h_3} \left[\frac{\partial(F_3 h_3)}{\partial u_2} - \frac{\partial(F_2 h_2)}{\partial u_3} \right] + \frac{\mathbf{a}_2}{h_3 h_1} \left[\frac{\partial(F_1 h_1)}{\partial u_3} - \frac{\partial(F_3 h_3)}{\partial u_1} \right] + \frac{\mathbf{a}_3}{h_1 h_2} \left[\frac{\partial(F_2 h_2)}{\partial u_1} - \frac{\partial(F_1 h_1)}{\partial u_2} \right]$$

$$\text{div } \mathbf{F} = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial(F_1 h_2 h_3)}{\partial u_1} + \frac{\partial(F_2 h_1 h_3)}{\partial u_2} + \frac{\partial(F_3 h_1 h_2)}{\partial u_3} \right]$$

$$\text{grad } f = \mathbf{a}_1 \frac{1}{h_1} \frac{\partial f}{\partial u_1} + \mathbf{a}_2 \frac{1}{h_2} \frac{\partial f}{\partial u_2} + \mathbf{a}_3 \frac{1}{h_3} \frac{\partial f}{\partial u_3}$$

In the Cartesian $h_1=h_2=h_3=1$

in spherical coordinates $h_1 = 1$, $h_2 = r$, and $h_3 = r \sin \theta$,

$h_1 = 1$, $h_2 = \rho$, $h_3 = 1$ Circular cylindrical

Dr.Tarig Algadey

Good Luck

Electromagnetic I الأسئلة المقترحة

Q1: (I) $z = 3 - j5$, Find $\ln(z)$

$$|z| = \sqrt{3^2 + 5^2} = 5.83, \theta = \tan^{-1} \frac{5}{3} = -59^\circ$$

$$z = |z| e^{j\theta} = 5.83 e^{-j59^\circ}$$

$$\begin{aligned} \ln(z) &= \ln(5.83 e^{-j59^\circ}) = \ln(5.83) + \ln(e^{-j59^\circ}) \\ &= 1.76 - j \frac{59^\circ}{180^\circ} \pi = 1.76 - j1.03 \end{aligned}$$

II) $z = \sqrt{3} e^{j\frac{3\pi}{4}}$

$$= \sqrt{3} \left(\cos \frac{3\pi}{4} + j \sin \frac{3\pi}{4} \right) = -1.22 + j1.22$$

$$= 1.22(-1 + j)$$

Q2) $A = a_x + 2a_y - 3a_z$ & $B = 2a_x - 4a_y$

$$|B| = \sqrt{4 + 16} = \sqrt{20}$$

I. $|A| = \sqrt{1 + 4 + 9} = \sqrt{14}$, $\hat{a}_A = \frac{a_x + 2a_y - 3a_z}{\sqrt{14}}$

II. $\theta_{AB} = \cos^{-1} \frac{A \cdot B}{|A||B|} = \frac{2 - 8}{\sqrt{14}\sqrt{20}} = \frac{-6}{\sqrt{14}\sqrt{20}}$

III. $(A \times a_y) \cdot a_z$

$$\begin{aligned} & \left((a_x + 2a_y - 3a_z) \times a_y \right) \cdot a_z = \\ & (a_z + 3a_x) \cdot a_z = \boxed{1} \end{aligned}$$

Q3 Find E_1 if $E_2 = 2a_x - 3a_y + 3a_z$ V/m

(Given that the x-y plane is the boundary between the two media, so

$$E_{1x} = E_{2x} = 2$$

$$E_{1y} = E_{2y} = -3$$

Exercice I and II

قوانين هيرتز

$$\text{Curl } F = \frac{a_1}{h_2 h_3} \left[\frac{\partial (F_3 h_3)}{\partial u_2} - \frac{\partial (F_2 h_2)}{\partial u_3} \right] + \frac{a_2}{h_3 h_1} \left[\frac{\partial (F_1 h_1)}{\partial u_3} - \frac{\partial (F_3 h_3)}{\partial u_1} \right] + \frac{a_3}{h_1 h_2} \left[\frac{\partial (F_2 h_2)}{\partial u_1} - \frac{\partial (F_1 h_1)}{\partial u_2} \right]$$

$$\text{Div } F = \frac{1}{h_1 h_2 h_3} \left[\frac{\partial (F_1 h_2 h_3)}{\partial u_1} + \frac{\partial (F_2 h_1 h_3)}{\partial u_2} + \frac{\partial (F_3 h_1 h_2)}{\partial u_3} \right]$$

$$\text{grad } f = a_1 \frac{1}{h_1} \frac{\partial f}{\partial u_1} + a_2 \frac{1}{h_2} \frac{\partial f}{\partial u_2} + a_3 \frac{1}{h_3} \frac{\partial f}{\partial u_3}$$

In the Cartesian, $h_1 = h_2 = h_3 = 1$, in spherical coordinates
 $h_1 = 1, h_2 = r, h_3 = r \sin \theta$

and $h_1 = 1, h_2 = \rho, h_3 = 1$ cylindrical.

$$\nabla \times E = -j\omega \mu H$$

$$\nabla \times H = J + j\omega \epsilon E$$

$$\nabla \cdot E = \rho / \epsilon$$

$$\nabla \cdot H = 0$$

For the z component

$$\epsilon_1 E_{1z} = \epsilon_2 E_{2z} \quad (\rho_s = 0)$$

$$E_{1z} = \frac{\epsilon_2}{\epsilon_1} E_{2z} = \frac{8\epsilon_0}{2\epsilon_0} \times 3 = \boxed{12}$$

$$\boxed{E_1 = 2ax - 3ay + 12az} \text{ V/m}$$

(Q4)

$$\rho = \rho^2 \sin\phi + 2\rho z$$

$$\nabla \rho = a_\rho \frac{\partial \rho}{\partial \rho} + a_\phi \frac{1}{\rho} \frac{\partial \rho}{\partial \phi} + a_z \frac{\partial \rho}{\partial z}$$

$$= a_\rho (2\rho \sin\phi + 2z) + a_\phi \frac{1}{\rho} \rho \sin\phi + 2a_z$$

$$\nabla \times \nabla \rho = a_\rho (0 - 0) + a_\phi (2 - 2) + a_z \left(\frac{1}{\rho} 2\rho \sin\phi - 2\rho \sin\phi \right)$$
$$= 0 + 0 + 0 = 0$$

~~divergence~~ $\nabla \times \nabla = 0$ مساوية مع الصفر